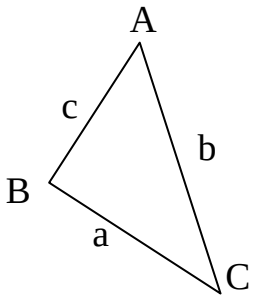
Day 5 – The Cosine Law

The **Cosine Law** relates sides and angles in a triangle in the following way:

$$a^2 = b^2 + c^2 - 2bc(\cos A)$$
$$b^2 = a^2 + c^2 - 2ac(\cos B)$$
$$c^2 = a^2 + b^2 - 2ab(\cos C)$$

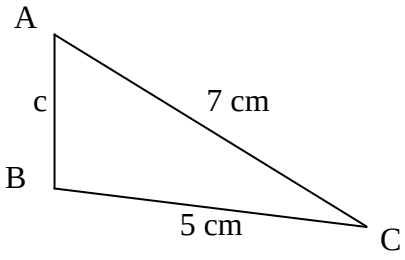


The Cosine Law can be used **only when you know**

1. 2 sides and the contained angle
2. all 3 sides

Using the Cosine Law to Find a Side

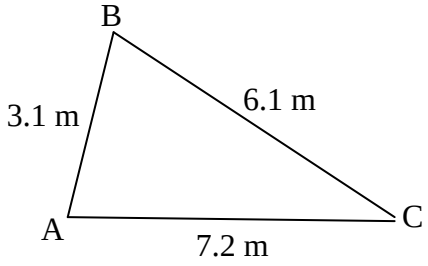
Example: Given ΔABC, find the length of the missing side, to one decimal place.



$$c^2 = a^2 + b^2 - 2ab(\cos C)$$
$$c^2 = 5^2 + 7^2 - 2(5)(7)(\cos 43)$$
$$c^2 = 25 + 49 - 70\cos 43$$
$$c^2 = 74 - 70\cos 43$$
$$c^2 \cong 22.8$$
$$c \cong \sqrt{22.8}$$
$$c \cong 4.8cm$$

Using the Cosine Law to Find an Angle

Example: Given ΔABC, find $< A$, to the nearest degree.

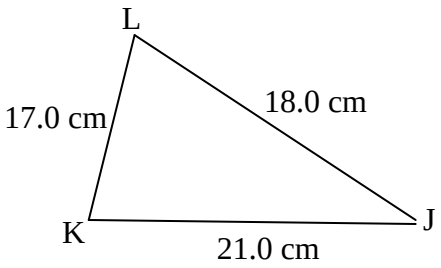


$$a^2 = b^2 + c^2 - 2bc(\cos A)$$
$$6.1^2 = 7.2^2 + 3.1^2 - 2(7.2)(3.1)(\cos A)$$
$$37.21 = 51.84 + 9.61 - 44.64\cos A$$
$$37.21 = 61.45 - 44.64\cos A$$
$$\cos A = \frac{37.21 - 61.45}{-44.64}$$
$$< A = \cos^{-1}\left(\frac{-24.24}{-44.64}\right)$$
$$< A \cong 57^o$$

Using the Cosine Law to Solve a Triangle

Example: Sketch and solve each triangle. Round your answers to the nearest degree and to the nearest tenth of a centimeter.

a) ΔJKL, with $j = 17.0cm$, $k = 18.0cm$ and $l = 21.0cm$.



Find $< J$.

$j^2 = k^2 + l^2 - 2kl(\cos J)$

$17.0^2 = 18.0^2 + 21.0^2 - 2(18.0)(21.0)(\cos J)$

$289 = 324 + 441 - 756\cos J$

$289 = 765 - 756\cos J$

$\cos J = \dfrac{289 - 765}{-756}$

$< J = \cos^{-1}\left(\dfrac{-476}{-756}\right)$

$< J \cong 51^o$

Find $< K$.

$k^2 = j^2 + l^2 - 2jl(\cos K)$

$18.0^2 = 17.0^2 + 21.0^2 - 2(17.0)(21.0)(\cos K)$

$324 = 289 + 441 - 714\cos K$

$324 = 730 - 714\cos K$

$\cos K = \dfrac{324 - 730}{-714}$

$< K = \cos^{-1}\left(\dfrac{-406}{-714}\right)$

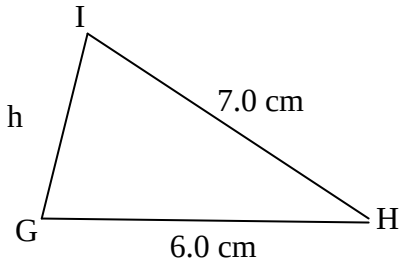
$< K \cong 55^o$

Find $< L$.

$< L \cong 180^o - 51^o - 55^o$

$< L \cong 74^o$

b) ΔGHI, with $< H = 43^o$, $g = 7.0cm$ and $i = 6.0cm$.



Find h.

$h^2 = g^2 + i^2 - 2gi(\cos H)$

$h^2 = 7^2 + 6^2 - 2(7)(6)(\cos 43^o)$

$h^2 = 49 + 36 - 84\cos 43$

$h^2 = 85 - 84\cos 43$

$h^2 \cong 23.6$

$$h \cong \sqrt{23.6}$$
$$h \cong 4.9cm$$

After finding side h, ΔGHI can be solved using the sine law or cosine law.

Using Sin Law:

Find $< G$.

$$\frac{h}{\sin H} = \frac{g}{\sin G}$$
$$\frac{4.9}{\sin 43} \cong \frac{7.0}{\sin G}$$
$$4.9 \sin G \cong 7.0 \sin 43$$
$$\sin G \cong \frac{7 \sin 43}{4.9}$$
$$< G \cong \sin^{-1}\left(\frac{7 \sin 43}{4.9}\right)$$
$$< G \cong 80^o$$

Find $< I$.

$$< I \cong 180^o - 43^o - 80^o$$
$$< I \cong 57^o$$

Using Cos Law:

Find $< G$.

$$g^2 = h^2 + i^2 - 2hi(\cos G)$$
$$7.0^2 = 4.9^2 + 6.0^2 - 2(4.9)(6.0)(\cos G)$$
$$49 \cong 23.6 + 36 - 58.3 \cos G$$
$$\cos G \cong \frac{49 - 23.6 - 36}{-58.3}$$
$$\cos G \cong \frac{-10.6}{-58.3}$$
$$< G \cong \cos^{-1}\left(\frac{-10.6}{-58.3}\right)$$
$$< G \cong 80^o$$

Find $< I$.

$$< I \cong 180^o - 43^o - 80^o$$
$$< I \cong 57^o$$